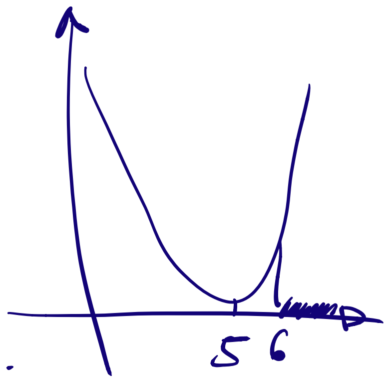
Examples

(1) $f_0(x) = (x-5)^2$

$f_1(x) = 6 - x$



(Aconvex) $\wedge$ (Aslater) holds.

KKT: (I)

$f_0'(x^*) + z_1 f_1'(x^*) = 0$

$\iff 2(x^*-5) - z_1 = 0$

(II) $z_1 f_1(x^*) = 0$, $z_1 \geq 0$

Case $z_1 > 0 \Rightarrow f_1(x) = 0 \Rightarrow x = 6$ ✓

Case $z_1 = 0 \Rightarrow 2(x^*-5) = 0$

$\Rightarrow x^* = 5$

but $f_1(x^*=5) = 1$ ↯

(2) $f_0(x) = (x-5)^2$

$f_1(x) = 3 - x$

KKT (I) $2(x^*-5) - z_1 = 0$

KKT (II) $z_1 \geq 0$:

Case $f_1(x^*) = 0 \Rightarrow x^* = 3$

$\Rightarrow 2(3-5) - z_1 = 0$

$\Rightarrow z_1 = -4$ ↯

Case $z_1 = 0 \Rightarrow x = 5$. ✓

(3) Hard margin case of SVM

In this case

$f_0(w) = \frac{1}{2}\|w\|^2$

$f_i(w) = 1 - y^{(i)} w \cdot x^{(i)}$ $\quad i = 1 \ldots m$

and no equality constraints

We know already

(Aconvex) holds

$\exists x^*$ optimal solution

and if training data lin. sep also (Aslater)

KKT (I)

$$\partial_w f_0(w^*) + \sum_{i=1}^{m} \alpha_i \partial_w f_i(w^*) = 0$$

$$\Rightarrow w^* = \sum_{i=1}^{m} \alpha_i y^{(i)} x^{(i)}$$

KKT (II) $\alpha_i f_i(w^*) = 0$ for $\alpha_i \geq 0$

KKT (III) empty

$\Rightarrow$ w^* determined by x_i for
i such that $f_i(w^*) = 0$
otherwise $\alpha_i = 0$!